Taylorovy polynomy

Radek Fučík

FJFI ČVUT v Praze

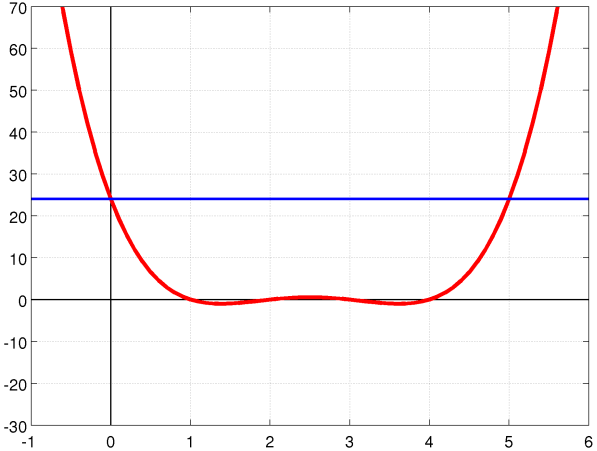
2024

Rozvoj polynomu v bodě $a = 0$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

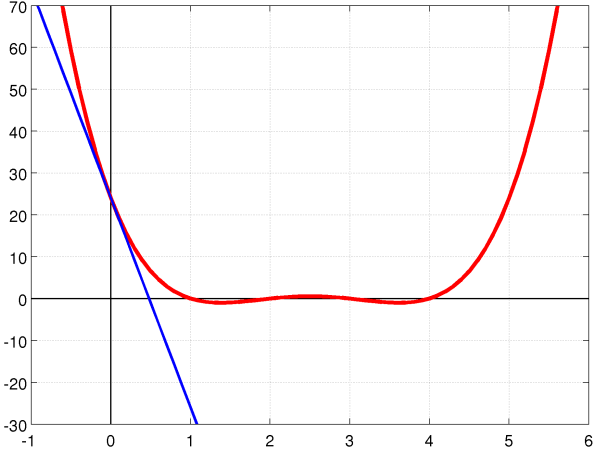
Rozvoj polynomu v bodě $a = 0$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_0(x) = 24$



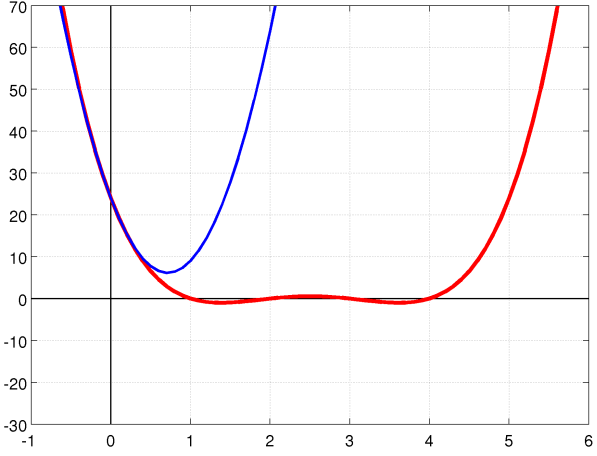
Rozvoj polynomu v bodě $a = 0$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_1(x) = 24 - 50x$



Rozvoj polynomu v bodě $a = 0$

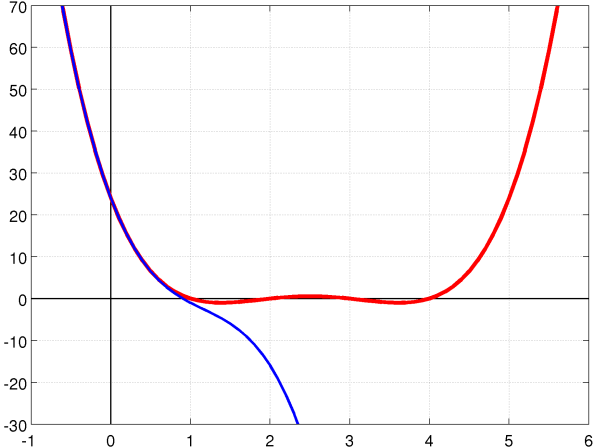
Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_2(x) = 24 - 50x + 35x^2$



Rozvoj polynomu v bodě $a = 0$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

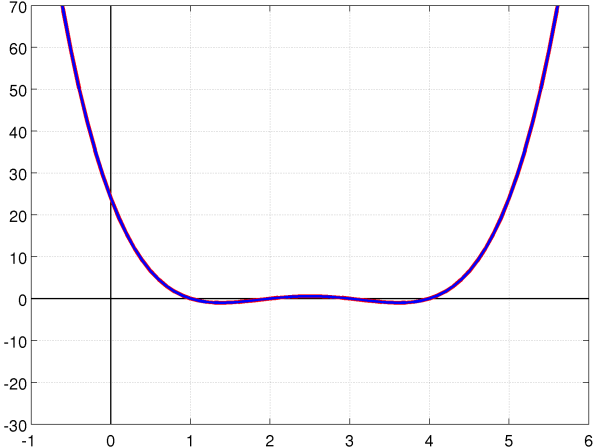
$T_3(x) = 24 - 50x + 35x^2 - 10x^3$



Rozvoj polynomu v bodě $a = 0$

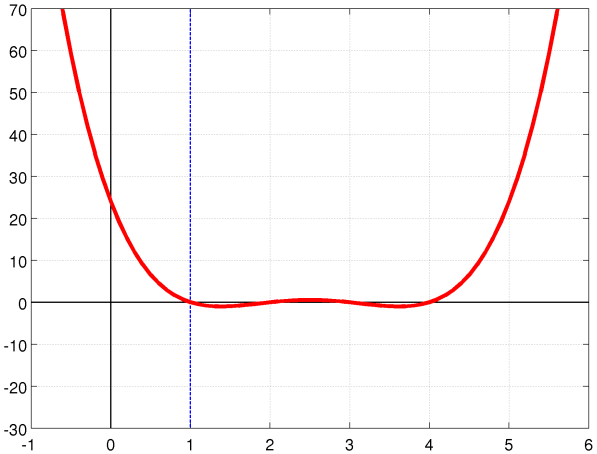
Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$T_4(x) = 24 - 50x + 35x^2 - 10x^3 + x^4 = p(x)$



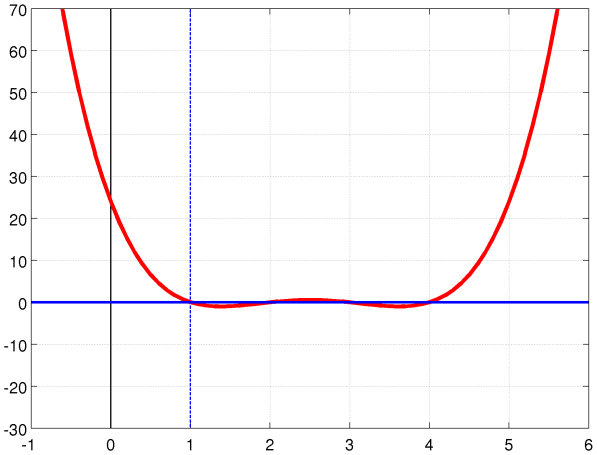
Rozvoj polynomu v bodě $a = 1$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$



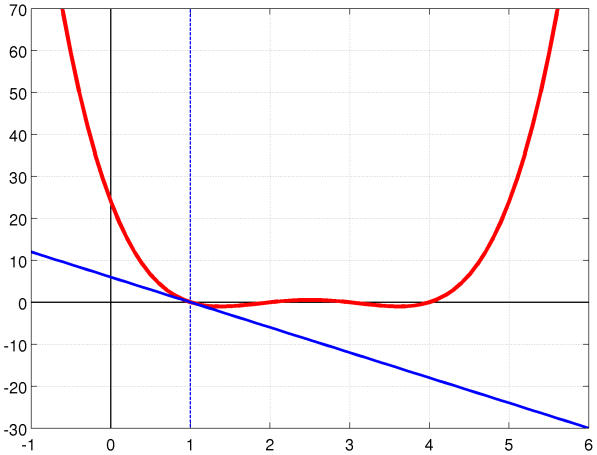
Rozvoj polynomu v bodě $a = 1$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_0(x) = 0$



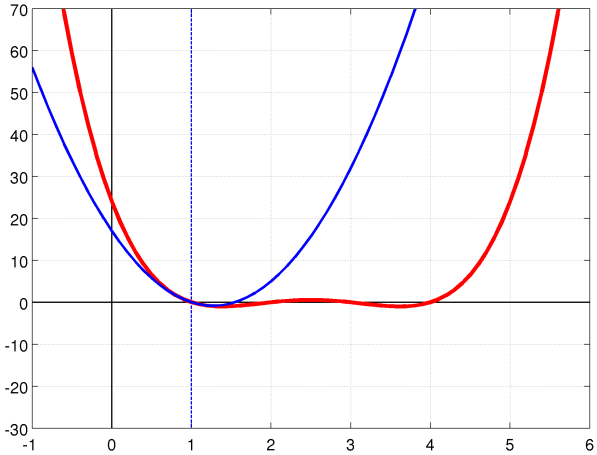
Rozvoj polynomu v bodě $a = 1$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_1(x) = -6(x - 1)$



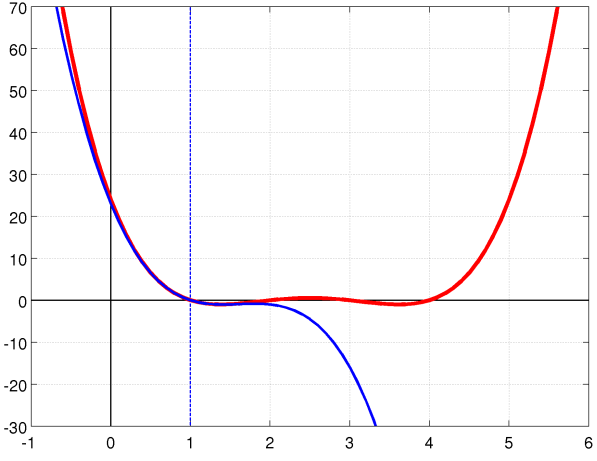
Rozvoj polynomu v bodě $a = 1$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_2(x) = -6(x-1) + 11(x-1)^2$



Rozvoj polynomu v bodě $a = 1$

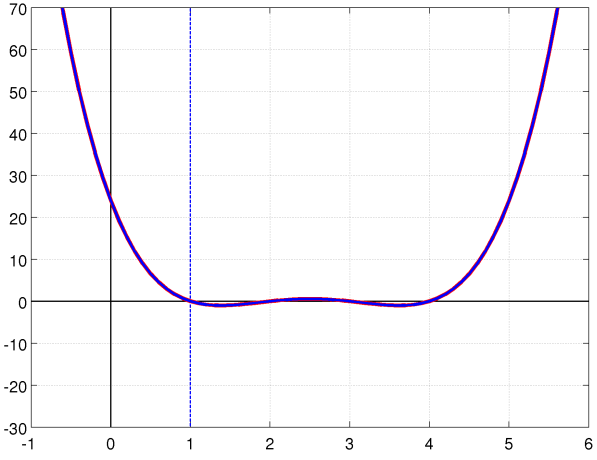
Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_3(x) = -6(x-1) + 11(x-1)^2 - 6(x-1)^3$



Rozvoj polynomu v bodě $a = 1$

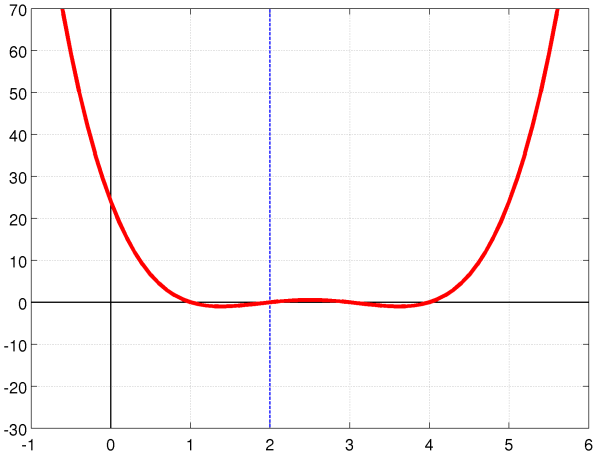
Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$T_4(x) = -6(x-1) + 11(x-1)^2 - 6(x-1)^3 + (x-1)^4 = p(x)$



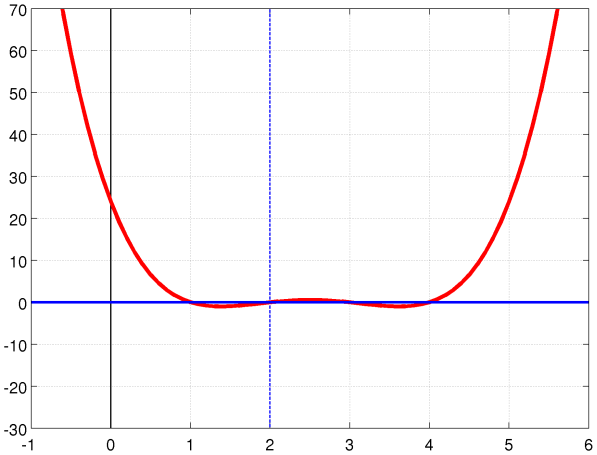
Rozvoj polynomu v bodě $a = 2$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$



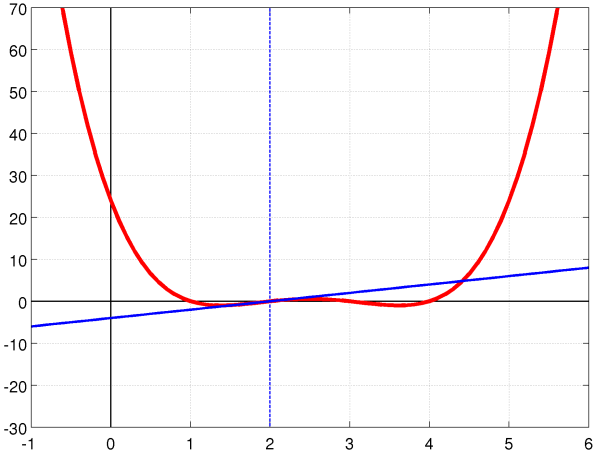
Rozvoj polynomu v bodě $a = 2$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_0(x) = 0$



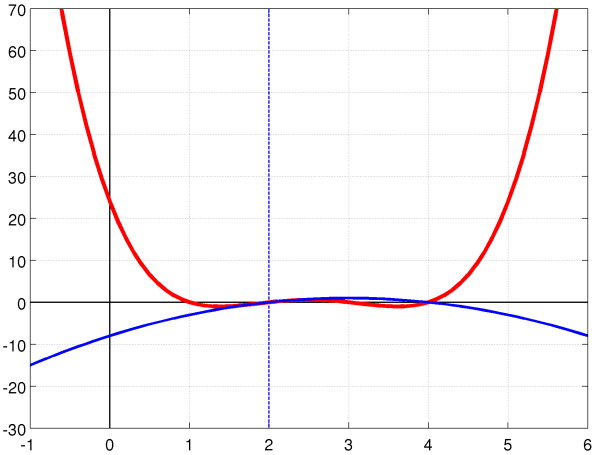
Rozvoj polynomu v bodě $a = 2$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_1(x) = 2(x - 2)$



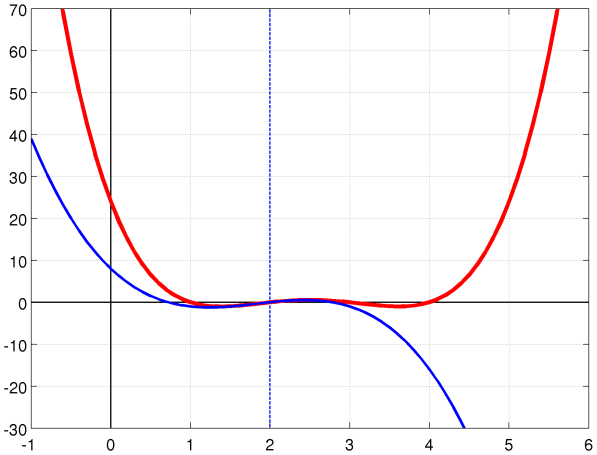
Rozvoj polynomu v bodě $a = 2$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_2(x) = 2(x-2) - (x-2)^2$



Rozvoj polynomu v bodě $a = 2$

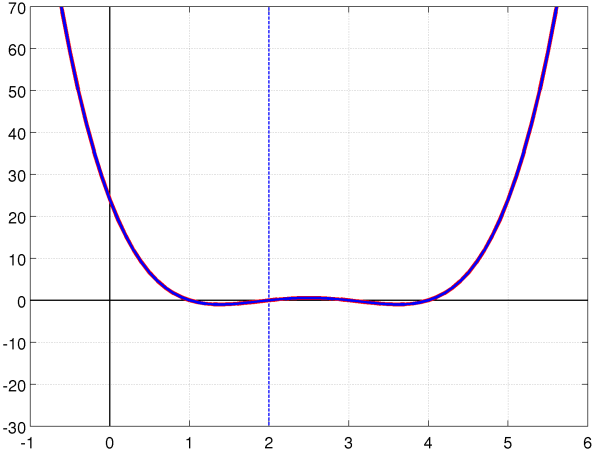
Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$T_3(x) = 2(x-2) - (x-2)^2 - 2(x-2)^3$



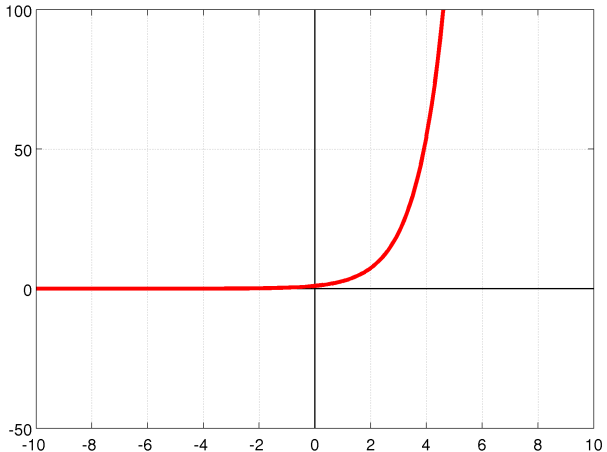
Rozvoj polynomu v bodě $a = 2$

Polynom $p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$T_4(x) = 2(x-2) - (x-2)^2 - 2(x-2)^3 + (x-2)^4 = p(x)$

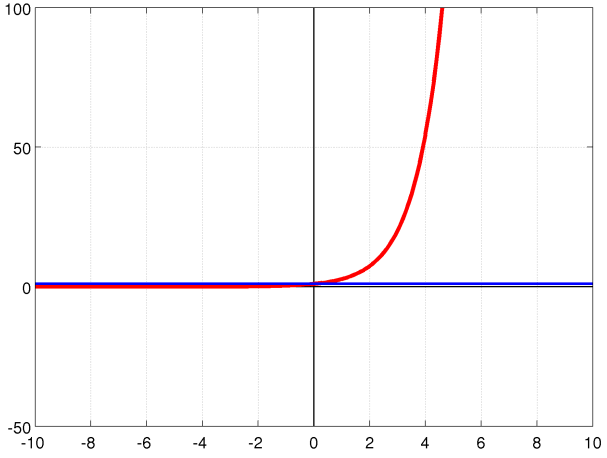


Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$



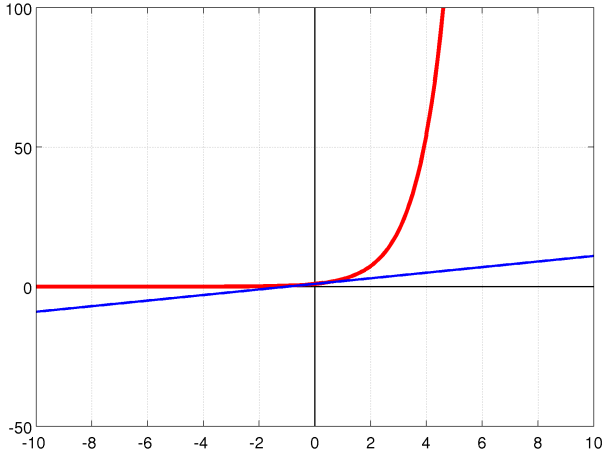
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$T_0(x) = 1$



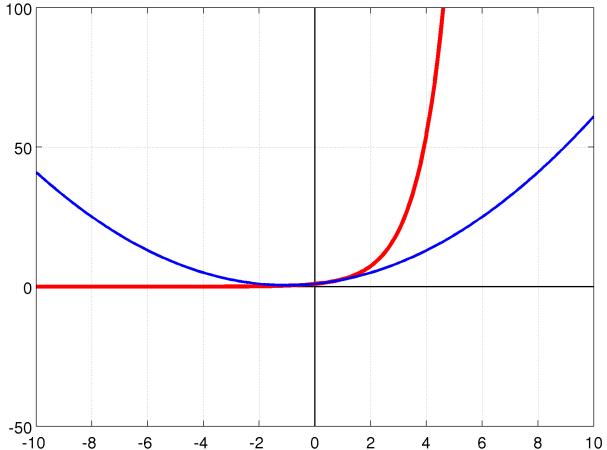
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$T_1(x) = 1 + x$



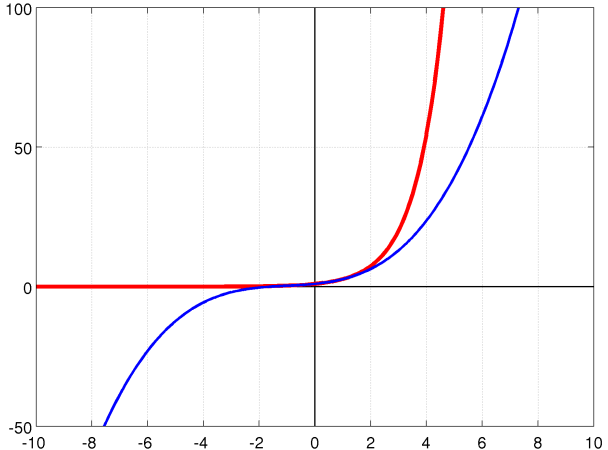
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$T_2(x) = 1 + x + \frac{1}{2!}x^2$



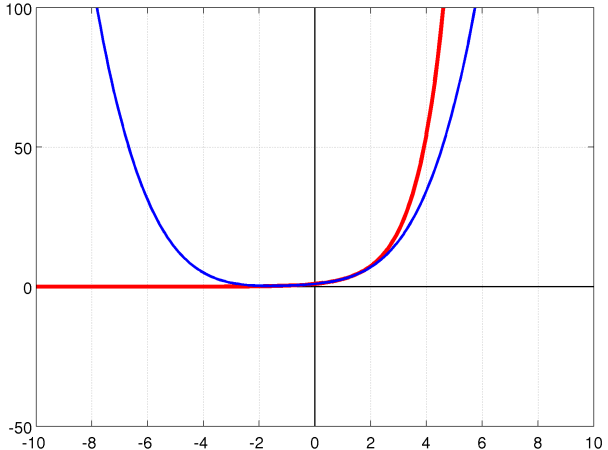
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$T_3(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$



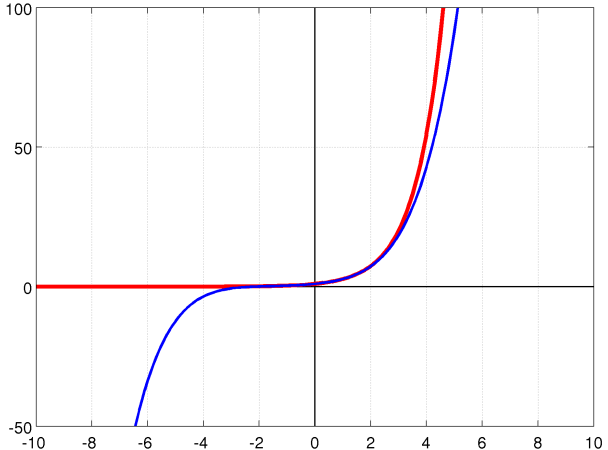
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$$T_4(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$$



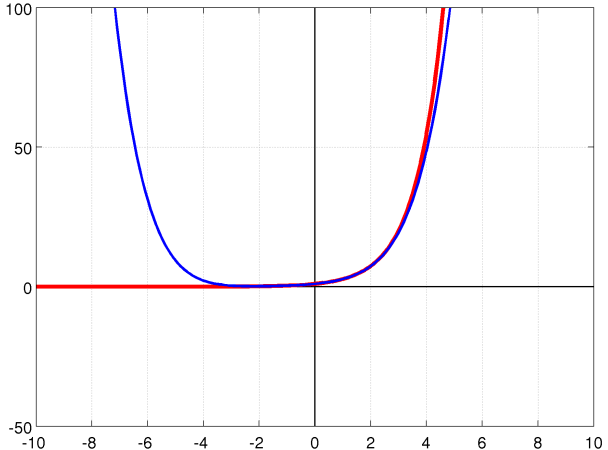
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$$T_5(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5$$



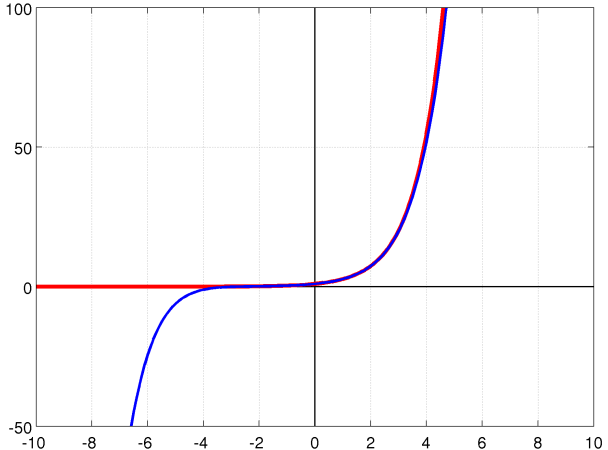
Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$$T_6(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6$$

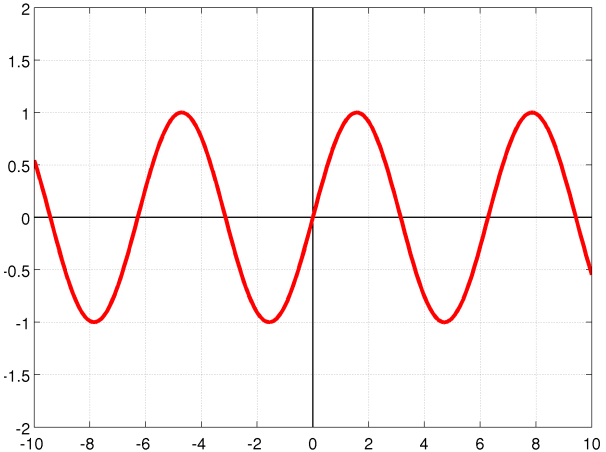


Rozvoj exponenciály $f(x) = \exp(x)$ v bodě $a = 0$

$$T_7(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7$$

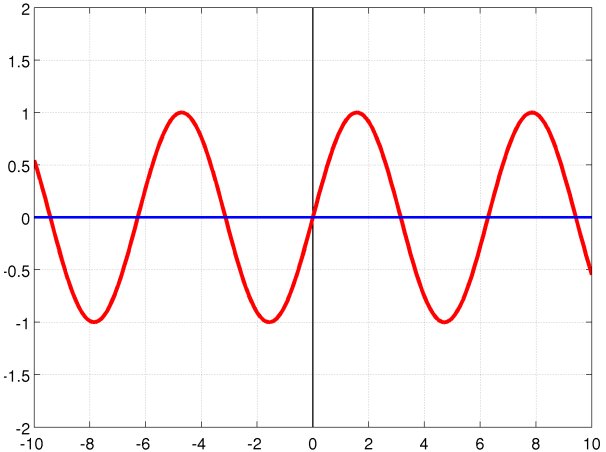


Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$



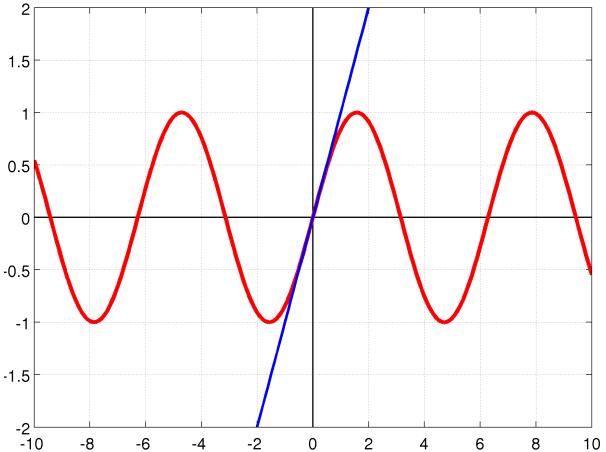
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_0(x) = 0$



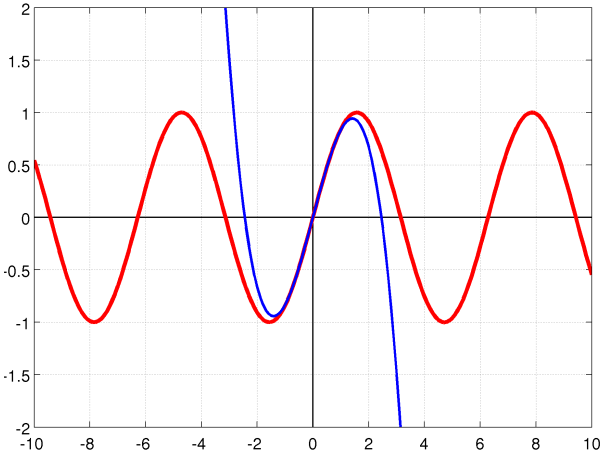
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_1(x) = T_2(x) = x$



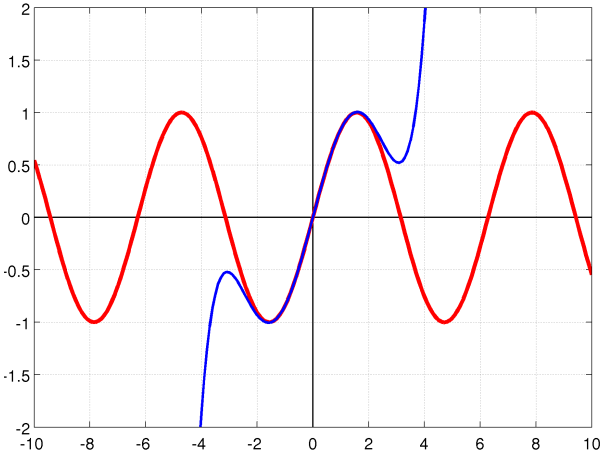
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_3(x) = T_4(x) = x - \frac{1}{3!}x^3$



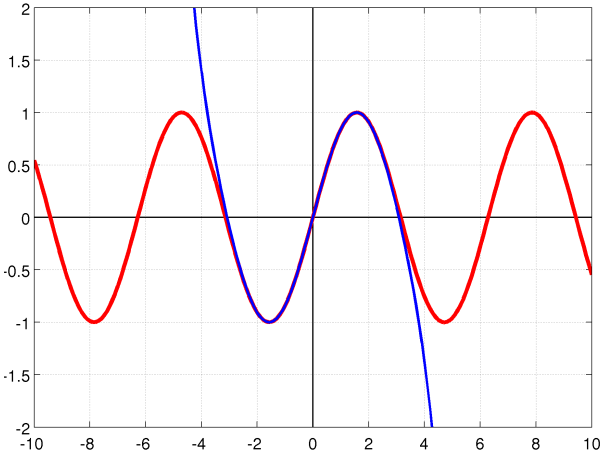
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_5(x) = T_6(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$



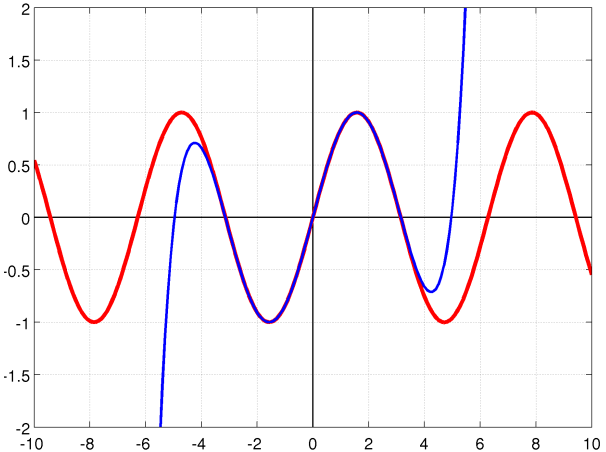
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_7(x) = T_8(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7$



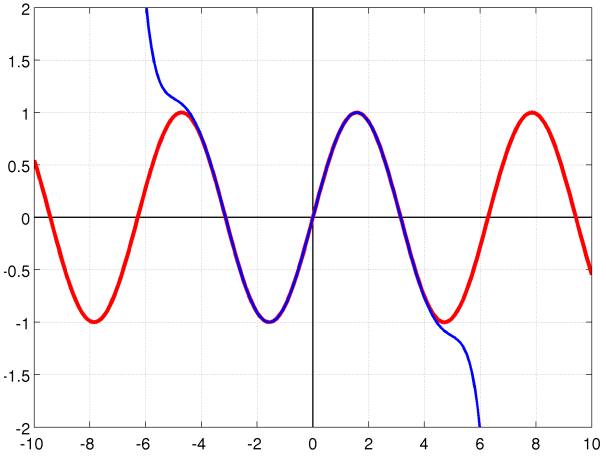
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$T_9(x) = T_{10}(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9$



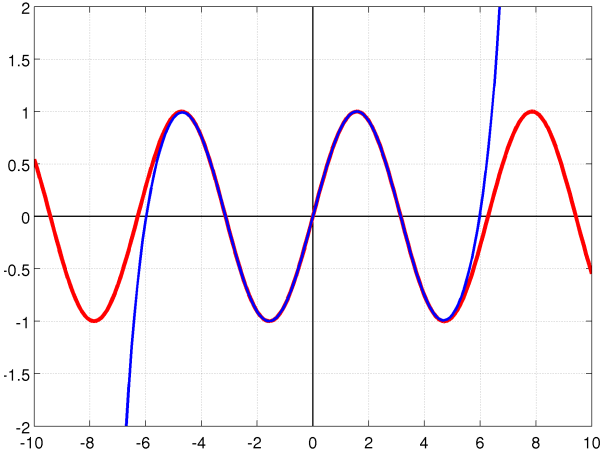
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{11}(x) = T_{12}(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11}$$



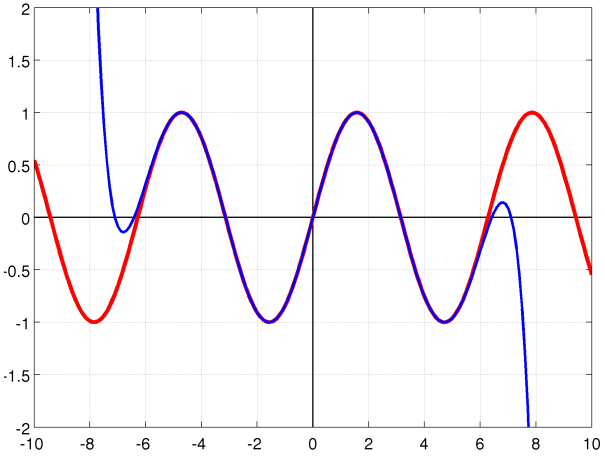
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{13}(x) = T_{14}(x) = \sum_{k=0}^{6} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$



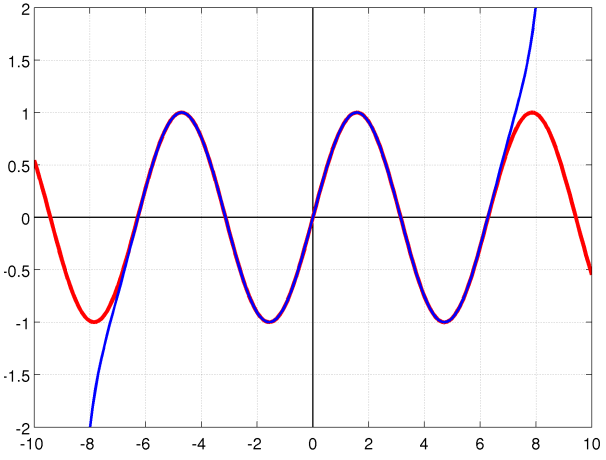
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{15}(x) = T_{16}(x) = \sum_{k=0}^{7} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$



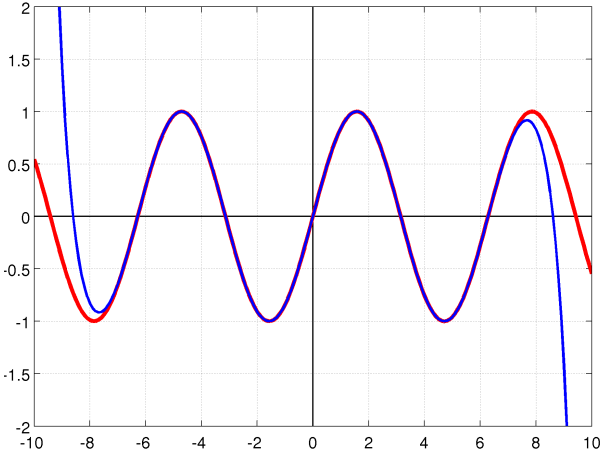
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{17}(x) = T_{18}(x) = \sum_{k=0}^{8} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$



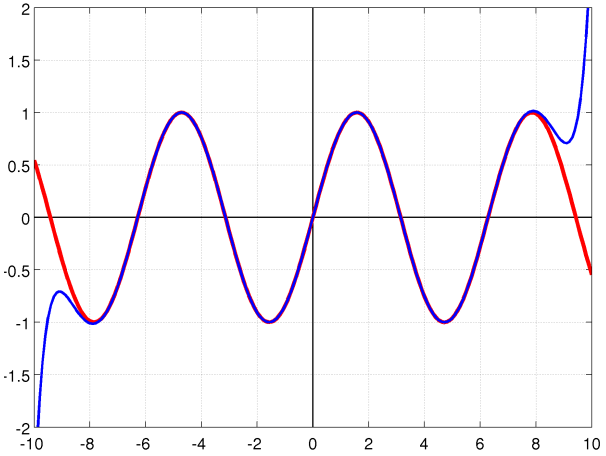
Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{19}(x) = T_{20}(x) = \sum_{k=0}^{9} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

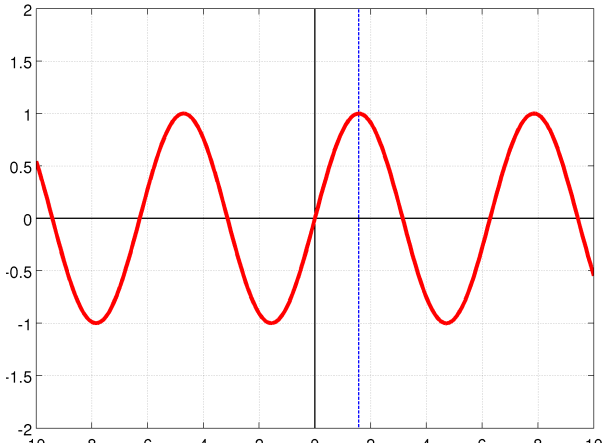


Rozvoj funkce $f(x) = \sin x$ v bodě $a = 0$

$$T_{21}(x) = T_{22}(x) = \sum_{k=0}^{10} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

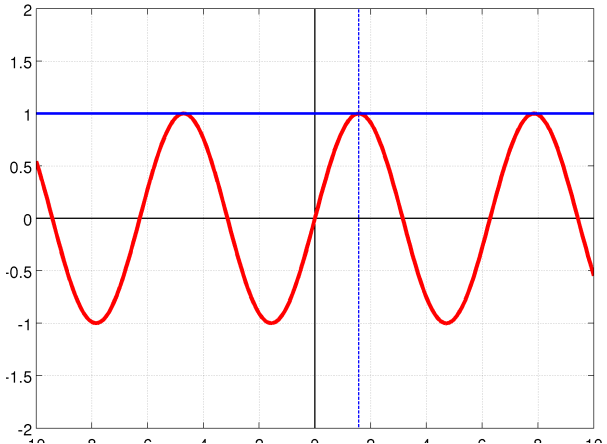


Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$



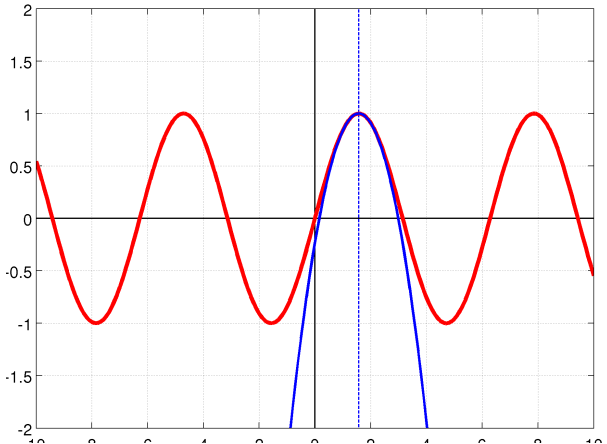
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$T_0(x) = 1 = \sin \frac{\pi}{2} = T_1(x)$



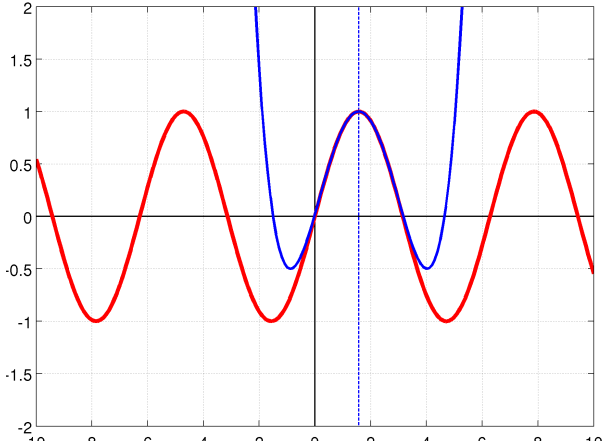
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_2(x) = T_3(x) = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2!}$$



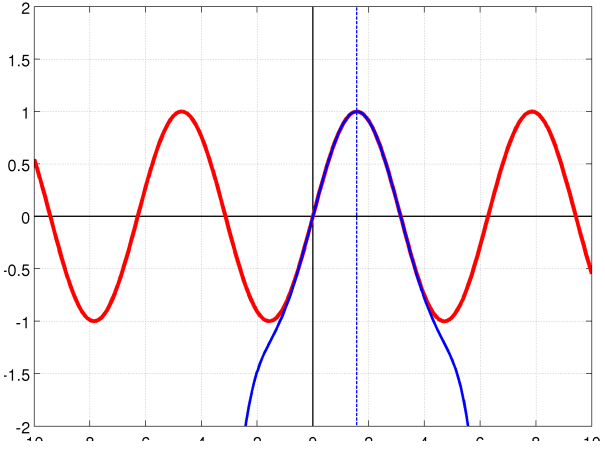
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_4(x) = T_5(x) = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2!} + \frac{\left(x - \frac{\pi}{2}\right)^4}{4!}$$



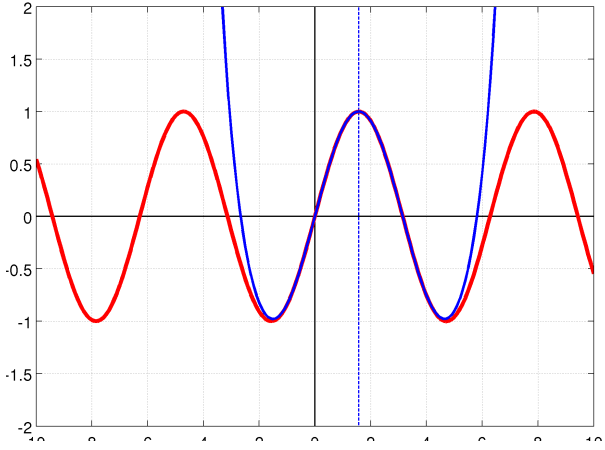
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_6(x) = T_7(x) = 1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} - \frac{\left(x-\frac{\pi}{2}\right)^6}{6!}$$



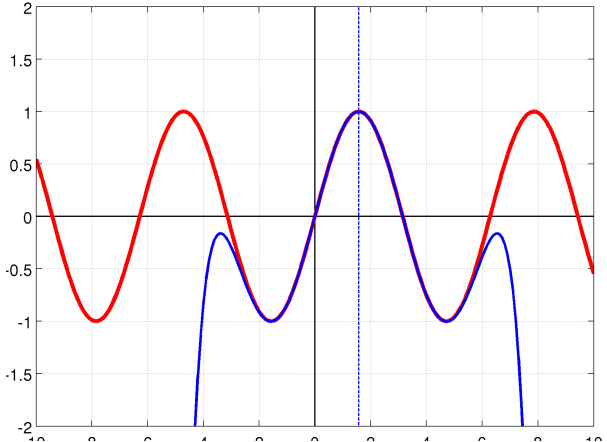
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_8(x) = T_9(x) = 1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} - \frac{\left(x-\frac{\pi}{2}\right)^6}{6!} + \frac{\left(x-\frac{\pi}{2}\right)^8}{8!}$$



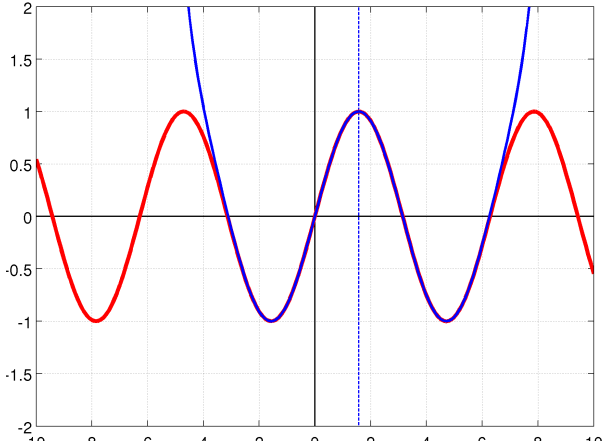
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{10}(x) = T_{11}(x) = \sum_{k=0}^{5} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



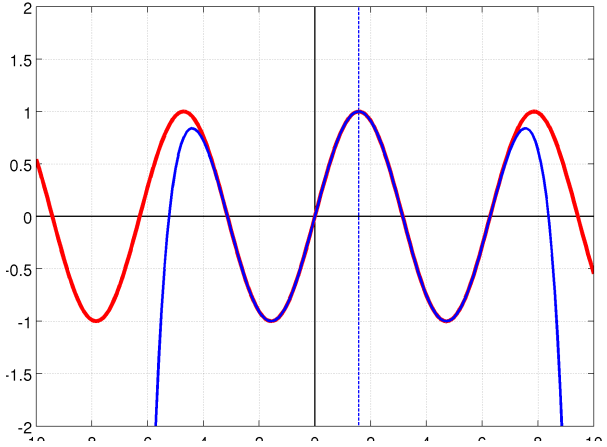
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{12}(x) = T_{13}(x) = \sum_{k=0}^{6} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



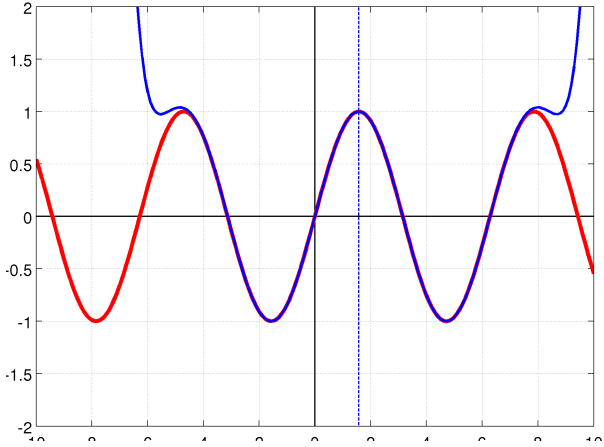
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{14}(x) = T_{15}(x) = \sum_{k=0}^{7} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



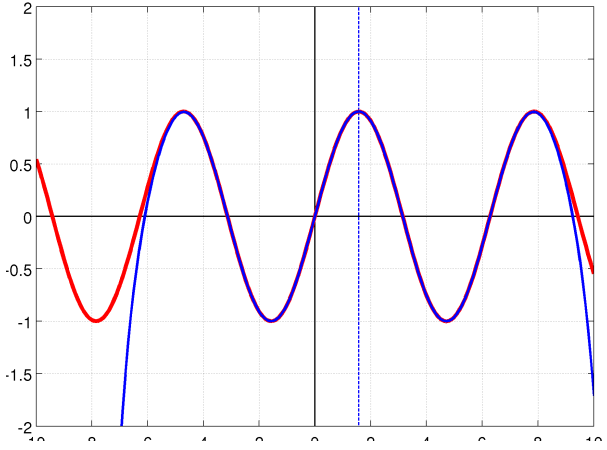
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{16}(x) = T_{17}(x) = \sum_{k=0}^{8} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



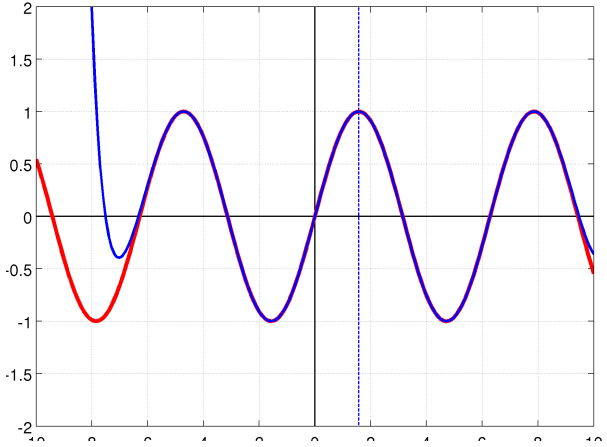
Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{18}(x) = T_{19}(x) = \sum_{k=0}^{9} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{20}(x) = T_{21}(x) = \sum_{k=0}^{10} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$



Rozvoj funkce $f(x) = \sin x$ v bodě $a = \frac{\pi}{2}$

$$T_{22}(x) = T_{23}(x) = \sum_{k=0}^{11} (-1)^k \frac{\left(x - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$