The best constant of discrete Sobolev inequality on regular polyhedron

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The best constant of discrete Sobolev inequality on regular polyhedron is obtained. The simplest case is the case of tetrahedron. The discrete Laplacian of it is the 4×4 matrix

$$A = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

Theorem *For any $u \in \mathbb{C}^4$ satisfying $\sum_{j=0}^{3} u(j) = 0$ it holds an inequality*

$$\left(\max_{0 \le j \le 3} |u(j)| \right)^2 \le C u^* A u$$

with suitable positive constant C. Among such C the best constant is $C_0 = 3/16$. If we substitute C by C_0 then the equality holds for $u = (3, -1, -1, -1)$, $(-1, 3, -1, -1)$, $(-1, -1, 3, -1)$, $(-1, -1, -1, 3)$.

For regular M-hedron the best constants $C_0(M)$ are as follows:

$$C_0(4) = 3/16, \quad C_0(6) = 29/96, \quad C_0(8) = 13/72,$$
$$C_0(12) = 137/300, \quad C_0(20) = 7/36.$$