The best constant of Sobolev inequality corresponding to clamped-free boundary value problem for $(-1)^M(d/dx)^{2M}$

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For $M = 1, 2, 3, \cdots$, we introduce Sobolev space

$$H = H(M) = \left\{ u(x) | u(x),\ u^{(M)}(x) \in L^2(-1,1),\ u^{(i)}(-1) = 0\ (0 \le i \le M-1) \right\}$$

Sobolev inner product

$$(u, v)_M = \int_{-1}^{1} u^{(M)}(x) \overline{v}^{(M)}(x) dx$$

and Sobolev energy

$$\|u\|_M^2 = \int_{-1}^{1} \left| u^{(M)}(x) \right|^2 dx.$$

$(\cdot, \cdot)_M$ is proved to be an inner product of H. H is Hilbert space with the inner product $(\cdot, \cdot)_M$.

Our conclusion is as follows.

Theorem *For any function $u(x) \in H$, there exists a positive constant C which is independent of $u(x)$ such that the following Sobolev inequality holds.*

$$\left(\sup_{|y| \le 1} |u(y)| \right)^2 \le C \int_{-1}^{1} \left| u^{(M)}(x) \right|^2 dx$$

Among such C the best constant C_0 is given by

$$C_0 = C(M) = \max_{|y| \le 1} G(y, y) = G(1, 1) = \frac{2^{2M-1}}{(2M-1)((M-1)!)^2}.$$

If we replace C by C_0 in Sobolev inequality, the equality holds for

$$u(x) = c\, G(x, 1) \qquad (-1 < x < 1)$$

for every complex number c.