Numerically repeated support splitting and merging phenomena in a porous media equation with strong absorption

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Numerical experiments to nonlinear diffusion equations suggest several interesting phenomena. One of them is the occurrence of *numerical support splitting phenomena* caused by strong absorption [1]. The most remarkable property is that the interaction between diffusion and absorption causes *numerically repeated support splitting and merging phenomena*. To realize such phenomena in $\mathbf{R}^1$ we consider the following model equations written in the form of the initial value problem and the initial-boundary value problem:

$$\text{(IVP)} \quad \begin{cases} v_t(t,x) = (v^m)_{xx} - cv^p \quad \text{on} \quad (0,\infty) \times \mathbf{R}^1, \\ v(0,x) = v^0(x) \quad \text{on} \quad \mathbf{R}^1, \end{cases} \tag{1}$$

$$\text{(IBP)} \quad \begin{cases} v_t(t,x) = (v^m)_{xx} - cv^p \quad \text{on} \quad (0,\infty) \times (-L,L), \\ v(t,-L) = f(t), \quad v(t,L) = g(t) \quad \text{for} \quad t \geq 0, \\ v(0,x) = v^0(x) \quad \text{on} \quad [-L,L], \end{cases} \tag{2}$$

where v denotes the density of the liquids, $m > 1$, $0 < p < 1$, $c > 0$, $m + p = 2$ and $v^0(x)(\geq 0)$.

For (IVP) we obtain

Theorem. *Let $N(\geq 2)$ be an arbitrary fixed integer. Then, it is possible to construct the initial function $v^0(x)$ for which N times repeated support splitting and merging phenomena* appear in (IVP).

For (IBP) we obtain some interesting phenomena by the numerical computation for the following cases with $m = 1.5$, $c = 6$ and $L = 1.5$,:

Case (I). $u^0(x) = 1.5$ and $\varphi(t) = 1.5$;
Case (II). $u^0(x) = 2.0$ and $\varphi(t) = 1.5 + 0.5\cos(2\pi t)$;
Case (III). $u^0(x) = 2.0$ and $\varphi(t) = 1.5 + 0.5\cos(12\pi t)$.
Here $u^0(x) \equiv (v^0(x))^{m-1}$ and $\varphi(t) \equiv f(t)^{m-1} \equiv g(t)^{m-1}$.

In this talk we introduce the numerical method and demonstrate some numerical examples for (IVP) and (IBP).

Reference

[1] T.Nakaki and K.Tomoeda, A finite difference scheme for some nonlinear diffusion equations in absorbing medium: support splitting phenomena, *SIAM J. Numer. Anal.*, **40**(2002), 945–964.