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BOUNDARY VALUE PROBLEMS IN BUILDING PHYSICS - THERMAL ENGINEERING OF BUILDINGS

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Abstract. The paper deals with applications of the parabolic partial differential equations with Newton boundary conditions in building physics. The stipulating of the temperature field in the domain - building construction or its part - stands obviously at the beginning of the solving of a task joined with temperature distribution and heat transfer related problems. Mathematical model of heat transfer comes out from Fourier law of heat conduction, the boundary conditions are indicated by Newton law of convection and simplified (linearized) Stefan - Bolzman law of radiation. Calculations of the temperature distribution especially on the inside surface of building envelope even while designing it, or before retrofitting it, enables us to predict the quality of inside environment in the rooms of the future building. In the first part of the paper there is an example, how to reach the required temperature distribution on the investigated surface - floor heating/cooling and its control. The second part is devoted to the impact of the inside surface temperature to the relative humidity near an observed surface in a specific room considering vapor production and ventilation, too.

Key words. Newton boundary conditions, Stefan-Bolzman law, heat conduction in buildings.

AMS subject classifications. 35K60, 35K65

1. Introduction. Determination of the temperature distribution in a domain is the essential calculation that stands at the beginning of the various tasks in building physics in civil engineering. In the both following problems the implicit method in time and the finite element method for the temperature distribution in the domain (2D section of the fraction of a building construction) in every time level are used.

Theory of energy transfer is a very important part of calculations during or even before designing and service - operation and control of the heating and cooling equipment in a building.

Recently, the amount of usage of radiant water based (hydronic) surface conditioning systems for heating and cooling purpose increased. Floor and ceiling radiant systems use a pipe coil placed close to the surface and an insulation layer guiding the heat flux the right direction. A thermo-active core system where pipes are embedded in the central part of a concrete structure utilizes both ceiling and floor heat transfer surfaces. Thus, the entire thermo-active building mass acts as an integrated buffer where heating/cooling energy can be stored [7].

2. Mathematical model and computer solution. We are investigating a 2D domain - a section of the floor with pipes while heating or cooling, see Fig. 2.1. For this purpose we have to solve the transient heat conduction equation

$$\rho c \frac{\partial T}{\partial t} = div(\lambda \nabla T), \tag{2.1}$$

in two dimensions where the notations are

ρ	density$[kg/m^3]$
c	specific heat capacity [J/(kg K)]

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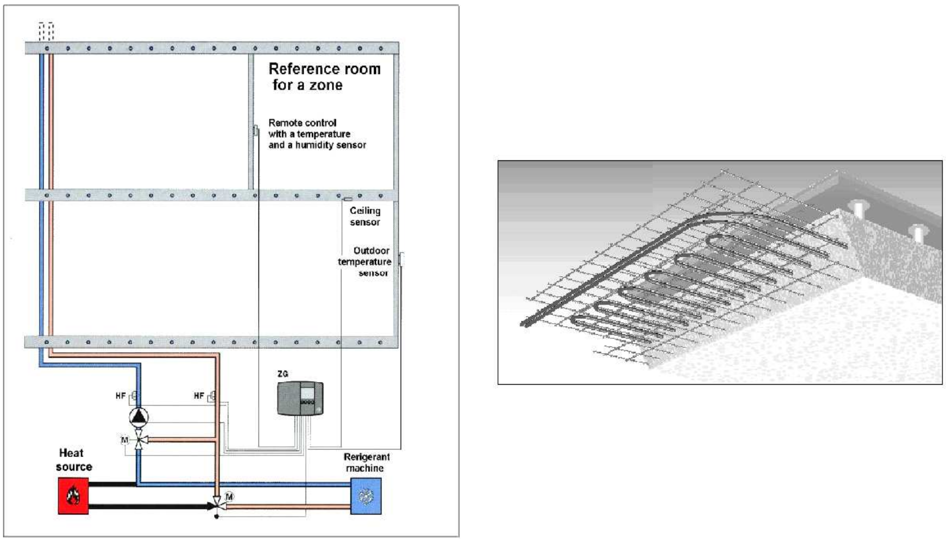


FIG. 1.1. *Floor heating system in a building*

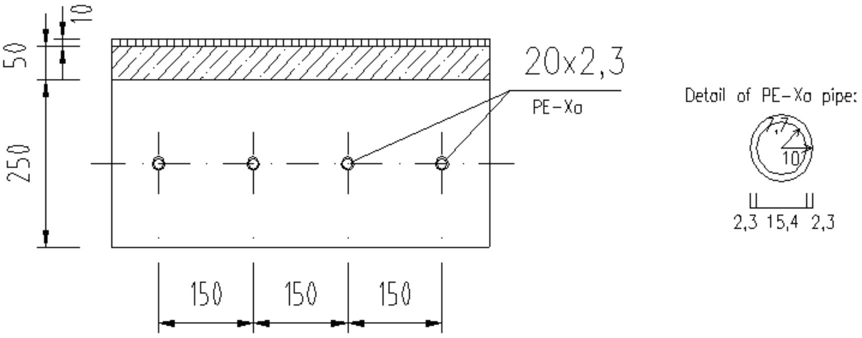


FIG. 2.1. *A representative part of the ceiling slab, the structure model and the dimension of pipes*

λ thermal conductivity coefficient [W/(m K)]
T temperature [K]
t time [s]

with the initial condition (initial temperature distribution) in the domain:

$$T(x, y, 0) = T_0(x, y). \tag{2.2}$$

The Newton boundary condition (convection) on the upper and lower (floor and ceiling) boundary of the domain:

$$q_c = h_c(T - T_a). \tag{2.3}$$

The rest of the mathematical model differs in the case of heating system being switched on or off:
When the circulation pumps are switched on, the water flowing through metal or plastic pipes keeps the temperature of their inside surface at the same value - we consider

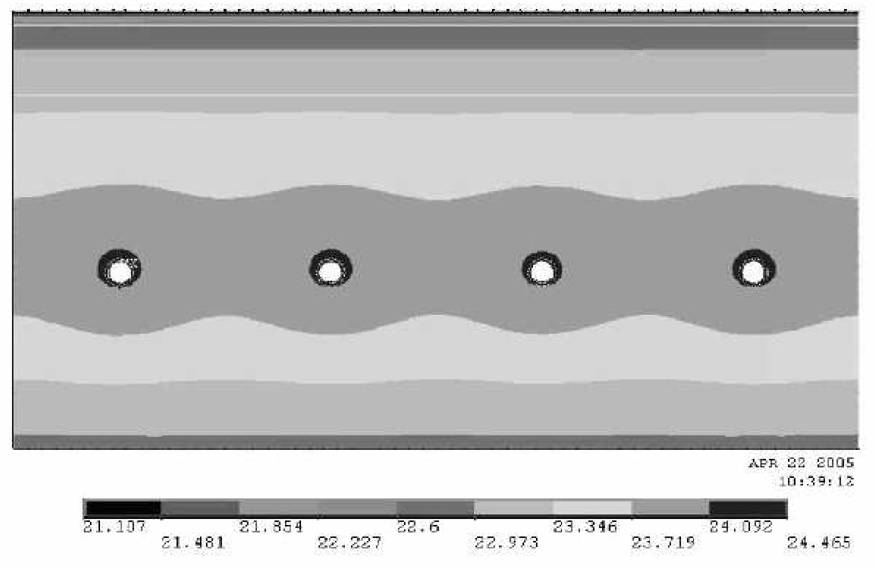


FIG. 2.2. *A temperature distribution in a slab in a time level, FEM software calculation*

Dirichlet boundary condition there. In the 2D section the pipes are represented by circles, see Fig. 2.1. We consider the rectangle without the circles as the investigated domain, where the heat transfer equation will be applied.

When the circulation pumps are switched off, the water in the pipes stops and becomes cooling what means that the heat transfer equation will be applied in the whole rectangle (circles representing pipes included). As the considered temperature range is not large, coefficient λ changes very slightly and we can use it as temperature independent, constant. Therefore we can rearrange (2.1) as follows:

$$\rho c \frac{\partial T}{\partial t} = \lambda \Delta T \ . \tag{2.4}$$

This calculation enables us to stipulate an optimal regime of switching of the heating system dependent on the composition of the floor, temperature of the water, etc.

3. Temperature distribution and hygric conditions. The most often places where moisture problems grow are called thermal bridges the places where low temperature and condensation can occur.

The lower temperature (deformation of the temperature field) on the inside surface is caused eventually by geometry, by composition (materials with different thermal properties used), or by both of them. Since the moisture situation (relative humidity) strongly depends on the temperature, we firstly stipulate temperature distribution in the thermal bridge using the same heat transfer equation (2.4) with the initial condition - initial temperature distribution in the domain (2.2) and with the Newton boundary condition - convection on the internal (indicated by index i) and external (index e) surface

$$q_{ci} = h_{ci}(T - T_{ai}), \tag{3.1}$$

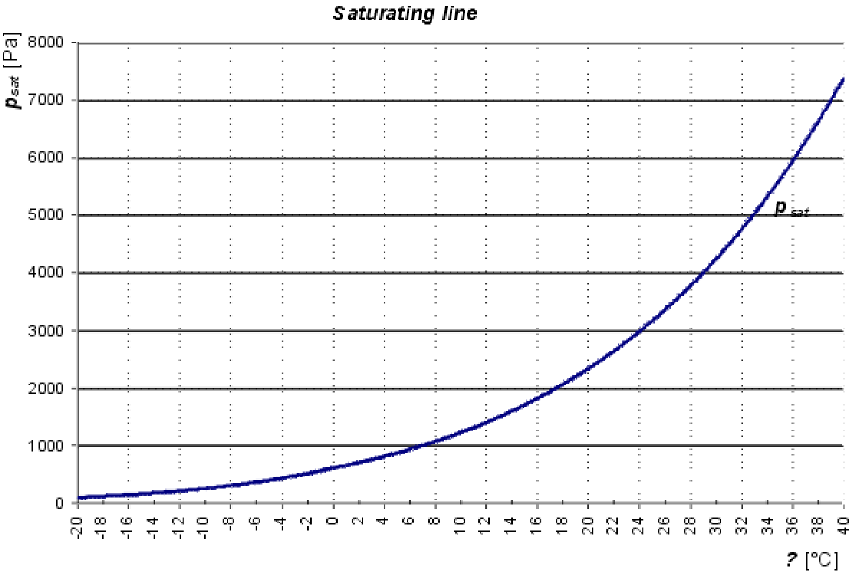


FIG. 2.3. *Saturation of the partial pressure depending on the temperature.*

$$q_{ce} = h_{ce}(T - T_{ae}). \tag{3.2}$$

At the places with the minimal temperature the risk of high humidity problems (wetting of walls, chemical changes on plaster, mould occurrence, static defects in worse cases) should be stipulated. For this purpose we use the hygric model. As shown in the Fig. 3.1a, the two places with locally minimal temperature have to be treated further by the hygric model avoiding the moisture problems.

4. Hygric model in a room. When we combine Gay-Lussac's law:

$$pV = mRT, \tag{4.1}$$

with the mass balance in the space

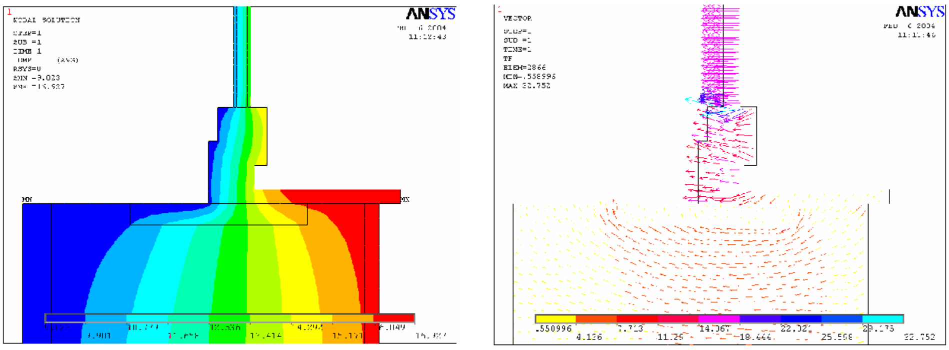
$$G + G_e = G_i + G_c, \tag{4.2}$$

we obtain a dependence of the partial vapor pressure on the several influencing parameters:

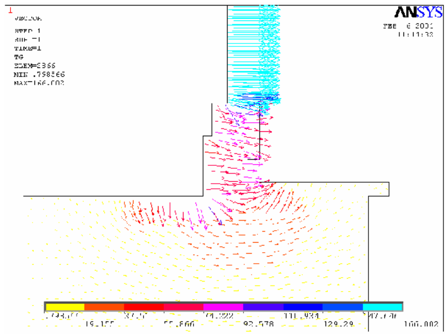
$$G + \frac{nVp_e}{RT_i} = \frac{nVp_i}{RT_i} + \frac{V}{RT_i}\frac{dp_i}{dt}, \tag{4.3}$$

with the notations

p	vapour pressure $[Pa]$
p_i, p_e	indoor, outdoor particular vapour pressure $[Pa]$
V	volume of the room $[m^3]$
m	mass $[kg]$
R	gas constant, for vapour R = 462 $[J/(kgK)]$
T, T_i	temperature, inside temperature $[K]$
G	vapour production $[kg/s]$



a) temperature b) thermal gradients



c) thermal fluxes

FIG. 3.1. *Figure shows: a) temperature distribution on the thermal bridge - window construction; b) thermal gradients in nodes (FEM discretization); c) thermal fluxes in nodes.*

G_e, G_i	amount of incoming / outcoming vapour by ventilation $[kg/s]$
G_c	amount of vapour consumed to partial pressure of vapour in the air $[kg/s]$
n	ventilation rate $[s^{-1}]$
t	time $[s]$

In the steady hygric state ($\frac{dp_{di}}{dt} = 0$), the solution of (4.3) is

$$p_i = p_e + \frac{GRT_i}{nV}. \tag{4.4}$$

In the transient hygric state the solution of (2.4) is:

$$p_i(t) = \frac{e^{-nt}(-RGT_i + Re^{nt}GT_i - np_e(t)V + e^{nt}p_e(t)V) + nVp_i(t_0)}{nV}. \tag{4.5}$$

By using the thermal model we obtain temperature distribution in the domain in the time range. In every moment of the time range we can compare indoor particular vapor pressure (4.5) with the saturating vapor pressure appropriated to the actual inside temperature. It is useful to do the comparison graphically, see [5].

5. Conclusions. It is very important to develop the interdisciplinary research that allows us to predict the quality of the internal environment of the future buildings even during designing it. The calculation done even during the designing of a new building can predict and prevent problems with high relative humidity on the surfaces and related problems - damages of materials and mould occurrence, etc. When we solve the problem with dampness in the building which is already built, the designer can propose suitable retrofitting (with foregoing calculation). They can be coped with by appropriate ventilation in the majority of the cases. As far as floor heating concerns (purposing, service, optimal use), the usage of differential equations and appropriate software is doubtless.

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